

Ballistic Lunar Transfer Design using the Deep Space Trajectory Explorer

Written by: Brian P. McCarthy, Jeremy Petersen, and Diane C. Davis



BALLISTIC LUNAR TRANSFER DESIGN USING THE DEEP SPACE TRAJECTORY EXPLORER

Brian P. McCarthy, Jeremy Petersen, and Diane C. Davis[‡]

In the coming years, numerous commerical companies and government agencies plan to expand their presence in cislunar space. Subsequently, an understanding of the cislunar gravitational environment is crucial to the success of these programs. Development of tools to effectively leverage natural dynamical structures helps streamline the trajectory design process. In this investigation, the functionality of the JavaFX-based Deep Space Trajectory Explorer (DSTE) is extended to construct ballistic lunar transfers to libration point orbits in the vicinity of the Moon.

INTRODUCTION

In 2020, NASA released the agency's lunar exploration program overview, providing Artemis and Gateway status reports as well as plans for additional extended lunar missions.¹ To enable such endeavors, an understanding of the cislunar gravitational environment is crucial to the success of the program. However, given the chaotic nature of a multi-body system, preliminary path planning in this environment is challenging. To meet these challenges, development of tools to streamline the preliminary trajectory design process that leverage dynamical structures in cislunar space is critical. Several tools have previously been developed to facilitate the early design process in this multibody regime. The Adaptive Trajectory Design (ATD) software facilitates construction of arcs in the circular restricted three-body problem (CR3BP) model to supply an initial guess to an ephemeris differential corrections process.^{2,3} The Poincare package was developed in JPL's MONTE software to aide in construction of itineraries in multi-body systems.⁴ Additionally, Generator and LTool have previously been used for multi-body trajectory design.^{5,6} The Deep Space Trajectory Explorer (DSTE) was developed as a JavaFX-based tool to aid in preliminary trajectory design in multi-body systems using interactive visualization techniques.^{7–10} In this investigation, the functionality of DSTE is extended to construct ballistic lunar transfers (BLTs) to cislunar libration point orbits.

Several previous, current, and planned missions are leveraging ballistic lunar transfer trajectories to reach the vicinity of the Moon. JAXA's Hiten spacecraft, KARI's KPLO mission, and NASA's GRAIL and CAPSTONE missions exploited BLT paths to successfully access to the lunar region, as well as ispace's HAKUTO-R mission, which is currently leveraging a BLT.^{11–15} This type of transfer offers a reduced propellant cost as an alternative to direct lunar transfer trajectories, but

^{*}Advanced Mission Design Engineer, a.i. solutions, Inc., 2101 E NASA Pkwy, Houston, TX 77058; brian.mccarthy@aisolutions.com

[†]Principle Systems Engineer, a.i. solutions, Inc., 4500 Forbes Blvd, Lanham, MD 20706; jeremy.petersen@ai-solutions.com

^{*}Aerospace Engineer, National Aeronautics and Space Administration, 2101 E NASA Pkwy, Houston, TX 77058; diane.c.davis@nasa.gov

typically requires a longer time of flight. Construction and characterization of BLTs have been investigated by several researchers previously. Parker and Anderson explore ballistic lunar transfers using dynamical systems and numerical methods within the context of a patched three-body model as well as an ephemeris model.¹⁶ Whitley et al. initially examined BLTs for uncrewed missions to the lunar Gateway.¹⁷ Parrish et al. survey ballistic lunar transfer options to NRHOs completely within the context of an ephemeris model.¹⁸ and examined operation considerations for BLTs to NRHOs.¹⁹ Additionally, McCarthy and Howell as well as Scheuerle and Howell investigate ballistic lunar transfers to periodic and quasi-periodic orbits within the context of a four-body model.^{20,21} This investigation leverages methodologies developed by previous researchers implemented in the DSTE to facilitate rapid construction of ballistic lunar transfers.

DYNAMICAL MODELS

There are two primary dynamical models used in this investigation, the CR3BP and the BCR4BP. These models are of medium fidelity and simultaneously include the gravity from more than one massive body. The fundamental motion within these models is constructed without introducing the complexity of an ephemeris model.

Circular Restricted Three-Body Problem

The CR3BP offers higher fidelity and additional behaviors in comparison to the two-body model. In this model, two gravitational bodies, denoted P_1 and P_2 , remain in circular Keplerian orbits about their mutual barycenter (i.e., center of mass). A third body, P_3 , moves under the gravitational influence of the two larger bodies and is assumed to be massless. The model is defined relative to a rotating coordinate system, where the $+\hat{x}$ direction is defined from the barycenter toward P_2 . The $+\hat{z}$ direction is defined parallel to the direction of the orbital angular momentum vector for P_1 and P_2 ; the \hat{y} direction completes the orthonormal triad. The position and velocity of P_3 relative to the barycenter in the rotating frame are defined as $\vec{x} = \begin{bmatrix} x & y & z & \dot{x} & \dot{y} & \dot{z} \end{bmatrix}^T$, where the first three and the last three elements are the position and relative velocity components, respectively. The equations of motion for a particle moving in the CR3BP are a set of three, second-order scalar differential equations of motion,

$$\ddot{x} - 2\dot{y} = \frac{\partial U^*}{\partial x} \tag{1}$$

$$\ddot{y} + 2\dot{x} = \frac{\partial U^*}{\partial y} \tag{2}$$

$$\ddot{z} = \frac{\partial U^*}{\partial z} \tag{3}$$

The pseudo-potential is a scalar defined solely as a function of position and the CR3BP mass parameter, $\mu = M_2/(M_1 + M_2)$, where M_1 and M_2 are the masses of P_1 and P_2 , respectively.²² The pseudo-potential function takes the following form,

$$U^* = \frac{x^2 + y^2}{2} + \frac{\mu}{r} + \frac{1 - \mu}{d}$$
(4)

where $d = \sqrt{(x+\mu)^2 + y^2 + z^2}$ and $r = \sqrt{(x-1+\mu)^2 + y^2 + z^2}$ represent the distances of P_3 relative to P_1 and P_2 , respectively. The CR3BP admits a single integral of the motion, commonly

denoted the Jacobi Constant (JC). The Jacobi Constant is a function of the pseudo-potential and the relative velocity magnitude expressed in the rotating reference frame,

$$JC = 2U^* - v^2 \tag{5}$$

where $v = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$. The Jacobi Constant is an energy-like quantity that characterizes motion in a CR3BP system and remains constant for all time over any ballistic arc propagated in the CR3BP. One advantage of the CR3BP model is that the system is time invariant. The CR3BP is a good approximation for a multi-body environment and the trajectory characteristics generally persist when transitioning results to a higher-fidelity ephemeris model.

Bicircular Restricted Four-Body Problem

The BCR4BP serves as a useful model for preliminary trajectory design where the complex dynamics in both the Earth-Moon and Sun-Earth regimes are significant. In this model, originally introduced by Huang, the gravitational forces from the Sun, Earth, and Moon are incorporated into a single framework while reducing the complexity as compared to an ephemeris model.²³ The motion of an infinitesimal mass (P_3) under the influence of three massive bodies, the Earth (P_1) , Moon (P_2) , and Sun (P_4) , is governed by the differential equations in the BCR4BP. The Earth and Moon move on circular paths about their mutual barycenter, denoted B_1 . Similarly, the Sun and B_1 move in circular, Keplerian motion about their mutual barycenter, denoted B_2 . In this formulation of the BCR4BP, the motion of the Earth and the Moon are not further perturbed by solar gravity, thus, the model is not coherent. Additionally, although not necessary, assuming that the Earth-Moon orbit plane is the same as the Sun- B_1 orbit plane is adequate for this analysis. The model is formulated in terms of either an Earth-Moon or Sun- B_1 rotating coordinate frame. For this investigation, the equations of motion are defined relative to an Earth-Moon rotating frame, where the $+\hat{x}$ -direction is defined from P_1 to the second primary, P_2 . The $+\hat{z}$ -direction is defined in the direction of orbital angular momentum for P_1 and P_2 ; the \hat{y} -direction completes the orthonormal triad. Additionally, this system is time dependent, where the location of the Sun in the Earth-Moon rotating frame is defined by a single angle, θ_S . The Sun moves in a clockwise direction around B_1 (i.e., θ_S is negative), as illustrated in Fig. 1(a). The equations of motion that describe the motion of the massless particle, P_3 , in the Earth-Moon rotating frame, are then defined,

$$\ddot{x} = 2\dot{y} + \frac{\partial\Upsilon}{\partial x} \tag{6}$$

$$\ddot{y} = -2\dot{x} + \frac{\partial\Upsilon}{\partial y} \tag{7}$$

$$\ddot{z} = \frac{\partial \Upsilon}{\partial z} \tag{8}$$

Note that Υ is the pseudo-potential in the BCR4BP as formulated in the Earth-Moon rotating frame. It is defined as,

$$\Upsilon = \frac{1-\mu}{r_{13}} + \frac{\mu}{r_{23}} + \frac{x^2 + y^2}{2} + \frac{m_4}{r_{43}} - \frac{m_4}{a_4^3}(x_4x + y_4y + z_4z)$$
(9)

where x_i , y_i , and z_i are the position components of P_i relative to the barycenter in the Earth-Moon rotating frame, μ is the Earth-Moon mass parameter, $\mu = \frac{M_2}{M_1 + M_2}$, r_{ij} is the position magnitude

of P_i relative to P_j , m_4 is the non-dimensional mass of P_4 , $m_4 = \frac{M_4}{M_1 + M_2}$, and a_4 is the semimajor axis of the circular orbit reflecting the Sun- B_1 motion. The term M_i is defined as the mass of P_i . Additionally, the nondimensional rotation rate of the Sun about B_1 , $\dot{\theta}_S$, is a function of a_4 and m_4 , $\dot{\theta}_S = \sqrt{\frac{1+m_4}{a_4^3}} - 1 = -0.9251986$. Similarly, motion viewed in a Sun- B_1 rotating frame is also advantageous. In this frame, the \hat{x}' -direction is oriented from the Sun to the Earth-Moon barycenter, B_1 . The \hat{z}' -direction is defined as the direction of the Sun- B_1 orbit angular momentum; the \hat{y}' -direction completes the triad. The Sun- B_1 rotating frame is illustrated in Fig. 1(b). It is useful to visualize motion in this frame to understand the influence of solar gravity on trajectories that include excursions beyond the Earth-Moon vicinity. Transformations of the states between the two rotating frames is straightforward.²⁴ The BCR4BP does not possess any integrals of the motion; however, an energy-like quantity, denoted the Earth-Moon Hamiltonian, is defined in the Earth-Moon rotating frame,

$$H = 2\Upsilon - v^2 \tag{10}$$

where Υ is the pseudo-potential function defined in Equation (9) and v is the relative velocity magnitude of the spacecraft in the Earth-Moon rotating frame, $v = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$. Properties of the Hamiltonian provide insight to effectively leverage ballistic lunar transfers, as noted by Scheuerle et al.²⁵ This formulation of the four-body problem offers a higher-fidelity environment than the CR3BP, but retains a reduced complexity, in contrast to an ephemeris model.



Figure 1. (a) Earth-Moon rotating frame and (b) $Sun-B_1$ rotating frame (right) as defined in the BCR4BP.

DEEP SPACE TRAJECTORY EXPLORER AND PERIAPSIS POINCARÉ MAPS

The DSTE leverages JavaFX for an interactive, visual approach to trajectory design in multibody systems. The user interface allows a trajectory designer to select primaries to define a CR3BP or BCR4BP system. Initial conditions are numerically integrated in these models, seamlessly incorporating multi-threading capabilities to distribute the computation of trajectory paths. Events are defined along the trajectory paths that define surfaces of section for the creation of interactive Poincaré maps. For example, surfaces of section defined by periapsis conditions are employed to generate periapsis Poincaré maps, fundamental to constructing ballistic lunar transfers in this investigation. There are two required conditions that define periapsis, or, more specifically, a point of closest approach along a path relative to a massive body. The first condition requires that the position vector of a state along a trajectory path relative to the primary be perpendicular to the velocity vector relative to the same primary; that is, the dot product between the position and velocity vectors is equal to zero,

$$\vec{r} \cdot \vec{v} = 0 \tag{11}$$

where \vec{r} is the position vector of the spacecraft realtive to a primary body and \vec{v} is the relative, rotating velocity vector of the spacecraft consistent with the differential equations formulated in the rotating frame. The first condition designates an apsis location along the trajectory, either periapsis or apoapsis. The second condition is derived to ensure that the derivative of the first condition is greater than zero,

$$v^2 + \vec{r} \cdot \vec{v} > 0 \tag{12}$$

where v is the magnitude of the spacecraft velocity and $\dot{\vec{v}}$ is the time derivative of the relative velocity as observed in the rotating frame, i.e., the relative acceleration. This second condition specifies that the first condition designates a periapsis, rather than an apoapsis point. The surface of section for the periapsis map is the appropriate hyperplane for assessment of these two conditions.

In addition to the generation of Poincaré maps, the DSTE provides the capability to compute families of multi-body periodic orbits. Members of the Earth-Moon L_2 northern halo family and distant retrograde orbit (DRO) family are computed in the CR3BP and rendered in Figures 2(a) and 2(b). A differential corrections and numerical continuations scheme are used to construct these families of orbits, as well as other families of orbits in an Earth-Moon CR3BP system within the DSTE. The DSTE also provides functionality to compute stable and unstable manifold trajectories as well as transfer arcs to and from periodic orbits. Periapsis maps, periodic orbit generation, and transfer generation functionality provide a foundation of tooling required to construct BLTs in the DSTE.



Figure 2. Subsets of the (a) Earth-Moon L_2 northern halo family and (b) the Earth-Moon DRO family computed in the DSTE.

CONSTRUCTING BALLISTIC LUNAR TRANSFERS

The framework for constructing initial guess solutions for ballistic lunar transfers to libration point orbits formulated by McCarthy and Howell as well as Scheuerle and Howell is implemented within the DSTE.^{20,21} As Scheuerle and Howell note, periodic orbits from the CR3BP coupled

with transfer arcs from the BCR4BP provide an initial guess of a BLT to transition to a higherfidelity ephemeris model. To begin the design process, a destination periodic orbit is selected in the DSTE. Since solar gravity is leveraged along a BLT path, the BCR4BP dynamics are incorporated to construct possible transfer paths in the DSTE. To find transfer paths, a process is developed in the DSTE. First the periodic orbit is discretized into a set of states. Each of these states represents a possible arrival location into the orbit. To explore possible paths at a variety of Sun-Earth-Moon geometries, each state is initialized at range of Sun angles as well. For example, in Figure 3(a), an L_2 northern halo orbit is discretized into 45 states. If 36 different Sun-Earth-Moon geometries are considered, a set of 1620 initial conditions is produced that are considered as possible arrival locations along the orbit. All of these initial conditions are propagated in reverse time for a userdefined time of flight, and the periapsis points relative to the Earth are computed along each path. Of the periapsis points recorded, only trajectories that have periapsis points within a certain radius of the Earth are recorded and stored, as illustrated in Figure 3(c). This process effectively isolates trajectories that, when propagated in forward time, produce transfers that depart the vicinity of the Earth and transit directly into the destination orbit. Furthermore, a range of Δv magnitudes and directions can also be incorporated into generating the set of initial conditions to increase the solution space for stable periodic orbits. This process can also be considered in forward time, producing paths that return a spacecraft from an orbit to the vicinity of the Earth.



Figure 3. (a) Destination orbit is discretized into a set of states. (b) States are propagated in reverse time at a range of Sun angles. (c) Only trajectories that possess periapsis radii close to the Earth are recorded.

The Batch Transfer Tool interface in the DSTE allows a user to effectively execute this process of generating transfers. As an example, consider a member of the northern L_2 halo family computed in the Earth-Moon CR3BP, which is plotted in the Earth-Moon rotating frame in Figure 4(a). The interface to this tool is rendered in Figure 4(b), where the red curve on the right side of the window represents a projection of the destination L_2 halo orbit in the $\hat{x}\hat{y}$ plane in the Earth-Moon rotating frame. A set of options and filters in the pane on the left side that allows the user to specify which initial conditions along the orbit are propagated in reverse time (rendered as the green dots along the projection of the halo orbit). The options include the propagation time, a range of magnitudes and directions of the arrival Δv , and the range of Sun angles of each initial point. Each of these initial conditions is numerically integrated for the propagation time specified by the user, exploiting native multi-threading capabilities provided by JavaFX. In this example, 23580 initial conditions are numerically integrated and the perigee points are recorded along each of the resulting trajectories. Additionally, a set of filters allows the user to specify which periapses satisfy desired properties. The filters include,

- Escape Distance: Distance calculated dynamically as the trajectory is propagated from the barycenter of the system. If the distance is greater than the Escape Distance, the numerical integration halts. This filter is useful to reduce the overall computation time by removing trajectories that will likely not return to the Earth-Moon vicinity without having to propagate for the full time of flight.
- Number of Periapses: The number specifying how many periapses relative to the Earth are encountered before the trajectory integration is terminated. By filtering trajectories with large numbers of periapses, solutions with excessive times of flight are excluded.
- Min/Max Radius: Desired radius bounds relative to the Earth. Trajectories that possess periapses radii greater than the maximum or less than the minimum are discarded. By bounding the minimum and maximum perigee radius to near-Earth space, it isolates transfers that produce a path directly from Earth to the destination orbit.
- Remove Prograde/Retrograde Checkboxes: Evaluates the \hat{z} -component of the cross product between the periapsis radius vector and velocity vector. If "Remove Prograde" is selected, then trajectories with a positive \hat{z} component are removed and "Remove Retrograde" is selected, then trajectories with a negative \hat{z} component are removed. Filtering out retrograde (or prograde) trajectories isolates transfers that are more realistic from a launch perspective.
- Limit: The maximum number of periapses to be evaluated over all propagations. All propagations stop when this limit is reached. When RAM is limited, this filter stops further generation of periapsis and trajectory data.

Trajectories are evaluated during runtime to determine if they satisfy the desired filters. Trajectories that include periapses that do not satisfy the filters are removed during runtime from the list of potential transfers. To isolate BLT trajectories, the minimum radius is set to the nominal radius of the Earth, 6378 km, and the maximum radius is set to 20,000 km to ensure that trajectories with periapses near Low Earth Orbit (LEO) are not discarded. This setup simulates a transfer trajectory with an initial state immediately after a translunar injection (TLI) maneuver from a parking orbit. The "Remove Retrograde" box is checked such that only prograde periapses near the Earth are included, to simulate a more realistic departure geometry. Lastly, the number of periapses is set

to unity, to isolate transfers that have shorter times of flight. Since the L_2 halo orbit selected as the destination is an unstable orbit, it possesses stable manifold trajectories that asymptotically approach the orbit. Thus, no insertion maneuver is imparted in DSTE when generating the BLTs to this orbit. However, recall that the L_2 halo orbit is computed in the CR3BP, and the dynamics used for generating the transfers are in the BCR4BP, similar to the technique by Scheuerle and Howell.²⁰ Consequently, this method uses two different models, where the interface between the models is at the insertion at the destination orbit. After all the initial conditions are integrated and filtered, a Earth-centered periapsis map is created, as illustrated in Figure 5(a) in the view with perigee points that satisfy the criteria specified in the tool. Additionally, these trajectories are rendered in the list in Figure 5(b), all representing potential transfers directly from Earth to the destination halo orbit. Note that the DSTE allows the user to also place the cursor over any of the transfers to obtain information about the transfers, such as the magnitude and direction of the insertion Δv and the instantaneous Jacobi Constant.



Figure 4. (a) Destination Earth-Moon L_2 southern halo orbit, rendered in the Earth-Moon rotating frame. (b) Selection of points along orbit are used as potential insertion locations from ballistic lunar transfer.



Figure 5. (a) P1-centered periapsis map showing perigee points associated with potential ballistic lunar transfers to an unstable L_2 halo orbit. (b) Filtered ballistic lunar transfers that provide direct access to the destination L_2 halo orbit.

Transfers to Stable and Nearly Stable Orbits

Cislunar orbits possess a wide variety of characteristics that make them desirable for various mission scenarios. In the previous example, the destination orbit is an unstable L_2 halo orbit. However, stable and nearly stable orbits possess characteristics that are desireable for human spacecflight. Namely, the Artemis 1 mission recently operated in a DRO for a half-revolution during the first Orion test flight beyond the Moon, and the Gateway spacecraft is planning to operate in a 9:2 synodic resonant L_2 Near Rectilinear Halo Orbit (NRHO).²⁶ DROs are known to be stable orbits, and thus, do not possess any stable or unstable manifolds to asymptotically approach or depart the orbit. Furthermore, the Gateway's NRHO is only slightly unstable, and the stable/unstable manifolds are not useful for transfer generation to/from the orbit. The DSTE provides features that make the design process straightforward for generating ballistic lunar transfers to these types of orbits as well. Consider a DRO that is of similar size to the Artemis 1, with a perilune radius of approximately 70,000 km. To generate transfers to this DRO in the DSTE, insertion maneuvers are required since the DRO is a stable orbit. Insertion maneuvers are imparted with a range between 1 m/s and 100 m/s in the velocity direction and a periapsis map relative to the Earth is created. By setting the mininum and maximum periapsis radius filters to 6378 km and 20000 km, respectively, the perigee map is rendered in the Earth-Moon rotating frame in Figure 6(a). The trajectories associated with the points on this map are also rendered in Figure 6(b), which shows that that there are numerous options and trajectory geometries that reach this DRO. The transfer trajectory boxed in blue in Figure 6(b) requires a Δv of 100 m/s to insert into the DRO and the trajectory boxed in red in Figure 6(b) leverages an outbound lunar flyby and requires an insertion Δv of 61 m/s. These trajectories along with the destination DRO are rendered in Figures 7(a) and 7(b). Note that the Earth and Moon are scaled larger than their actual size in the 3D views.



Figure 6. (a) P1-centered periapsis map showing perigee points associated with potential ballistic lunar transfers to a lunar DRO. (b) Transfer trajectory options to DRO associated with points from perigee map.

The BLT generation process is repeated to design transfers to a 9:2 synodic resonant southern L_2 NRHO, the orbit chosen for NASA's lunar Gateway station. The orbit is considered unstable, but the hyperbolic manifolds associated with this NRHO depart prohibitively slowly. Thus, imparting a Δv to insert into the orbit is a more effective approach. Using the 9:2 NRHO as the destination orbit, a set of initial conditions is selected along the section of the orbit near perilune, since it has been previously demonstrated that insertion near apolune is typically requires a higher insertion Δv .¹⁸ The periapsis map along with the NRHO and initial conditions on the NRHO are displayed



Figure 7. Ballistic lunar transfers to a lunar DRO without (a) and with (b) a lunar flyby. The insertion Δv for each of these transfers is 100 m/s and 61 m/s, respectively.

in Figure 8. Note that on the right side of the map, perigee points exist that are outside the bounds of the 20,000 km maximum perigee radius filter. F In this scenario, the Number of Perigee points filter is set to 2. The points on the right side of Figure 8, mostly colored in shades of blue and purple, represent the first perigee points along the trajectories. The second perigee points associated with the backwards propagation of each trajectory appear on the left side of Figure 8, within the specified 20,000 km radius limit of the Earth. Two candidate BLTs to the 9:2 NRHO are rendered in the Earth-Moon rotating frame in Figures 9(a) and 9(b), where the first candidate requires a 71 m/s insertion maneuver and second candidate BLT includes an outbound lunar flyby, with a 66 m/s insertion Δv . The framework developed in the DSTE demonstrates the flexibility to generate initial BLT solutions to various cislunar destinations, regardless of stability properties.



Figure 8. Perigee map for transfers generated to 9:2 NRHO projected into the $\hat{x}\hat{y}$ plane of the Earth-Moon rotating frame.

TRANSITIONING TRANSFERS TO AN EPHEMERIS MODEL

The CR3BP and BCR4BP both offer useful insight into the dynamical behavior associated with ballistic lunar transfers and possible destination orbits; however, validation in a higher-fidelity model is an important step in the design process. There are several software packages that facilitate transition and optimization of higher-fidelity trajectories, such as Copernicus, GMAT, FreeFlyer, STK, EMTG and ATD.^{27–32} The DSTE focuses on constructing trajectories in medium fidelity models, such as the CR3BP and the BCR4BP, and serves a sandbox to explore and filter through



Figure 9. Two BLTs to the 9:2 synodic L_2 NRHO (a) without and (b) with an outbound lunar flyby.

the solution space during the preliminary design process. The solutions found in the DSTE then serve as initial guess trajectories to be transitioned to a higher-fidelity model. This investigation leverages the FreeFlyer commercial off-the-shelf software package for ephemeris propagation and transition to the ephemeris model. FreeFlyer's optimization capabilities provide a convenient way to formulate a corrections problem that transitions an initial guess obtained in the DSTE to the Sun-Earth-Moon ephemeris model.³³ The corrections problem is formulated as a multiple shooting scheme to satisfy a set of constraints. The first step in the transition process is to select an initial epoch corresponding to the Sun-Earth-Moon geometry associated with the initial guess found in the BCR4BP in the DSTE. The methodology to select this epoch is outlined by McCarthy and Howell.²¹ The next step is to discretize the trajectory into a set of nodes, or patch points. Each node is associated with a six-element state vector, an epoch, and a propagation time. In addition to discretizing the transfer trajectory, 10 revolutions of the destination orbit are discretized into nodes to ensure the geometry of the destination orbit is maintained. Lastly, the state vector associated with each node is transformed into the Moon-centered J2000 inertial frame. Scheuerle and Howell as well as Boudad demonstrate that certain nodes, depending on the distance from the Earth-Moon neighborhood, should be transformed from the Earth-Moon or Sun- B_1 rotating frame into the inertial frame.^{20,34} In this investigation, states that have a radius greater than 1,000,000 km relative to the Earth-Moon barycenter are transformed from the Sun- B_1 rotating frame into the Moon-centered J2000 frame and states that have a radius less than 1,000,000 km are transformed from the Earth-Moon rotating frame. By performing the transformation in this way, the geometry of initial guess obtained in the DSTE is maintained. Once the states are transformed into the inertial frame, a freevariable vector is constructed with the elements of the states of each node, the epoch of each node, and the time of flight associated with each node. State and epoch time continuity are constrained between successive nodes and the initial node is constrained to a 150 km altitude perigee to represent a post-TLI state.

In Figures 10, 11, and 12, the initial guesses from the DSTE, propagated in an ephemeris model in FreeFlyer, are rendered in three different frames. In Figure 10, a BLT to an unstable L_2 halo orbit is rendered, Figure 11 illustrates a BLT to a lunar DRO without an outbound lunar flyby, and Figure 12 illustrates a BLT to the 9:2 synodic L_2 NRHO without an outbound lunar flyby. The ephemeris model includes point mass modeling for the Sun, Earth, Moon, and all planetary bodies including Mercury, Venus, Mars, Jupiter, Saturn, Uranus, and Neptune. Celestial object state information is obtained from the DE430 planetary and lunar ephemeris.³⁵ Three views are included in each figure: the Earth-centered inertial MJ2000 frame, the Sun-B1 rotating frame, and the Earth-Moon rotating frame. Each patch point, designated by the white circles, is propagated forward in time in the ephemeris model, based on the discretized time of flight from the DSTE initial guess. The initial guesses shown in Figures 10, 11, and 12 highlight the impact of transitioning the trajectory from a simplified model into a higher-fidelity ephemeris model, as state continuity is lost between the end of a segment and the beginning of the next segment, particularly during the ballistic lunar transfer phase. The larger gaps in continuity between segments during the transfer phase show the different transition process for patch points above the 1,000,000 km threshold, i.e., transforming states from the Sun- B_1 rotating frame to the inertial frame. While the gaps in the initial guess trajectories appear large, there is a significant improvement of the initial guess for the inertial states as compared to transforming all patch points only from the Earth-Moon or Sun- B_1 rotating frame.







To converge the entire trajectory in the full ephemeris model, a two-step multiple-shooting process is utilized. First, the destination orbit is converged independently to ensure the geometry of





Figure 11. Initial guess for BLT to a lunar distant retrograde orbit in an ephemeris model, rendered in the (a) Earth-centered inertial frame, (b) Sun- B_1 rotating frame, (c) Earth-Moon rotating frame, and (d) zoomed in view of the DRO in the Earth-Moon rotating frame.

the destination orbit is maintained in the ephemeris model. In Figure 13, the initial guesses for the unstable halo orbit, DRO, and NRHO are rendered in the left column, and converged ephemeris trajectories in the ephemeris model appear in the right column. Recall that the initial guess from the DSTE includes 10 revolutions of the destination orbit. Note that the ephemeris solution in the right column shows the distortion effects on the shape of the periodic orbit when it is converged, but the geometry of the orbit is maintained. Once the periodic orbit has converged, the second step is to add the transfer phase to the multiple-shooting problem to solve for state continuity between the BLT and the periodic orbit and to constrain the initial condition at Earth to have a 150 km perigee altitude. In Figures 14, 15, and 16, the fully converged trajectories are rendered; they maintain a geometry similar the initial guess from Figures 10, 11, and 12. In all three transfer examples, a trajectory correction maneuver (TCM) modeled as a velocity discontinuity is allowed during the BLT near apogee to provide more flexibility during the corrections process. Utlimately, the unstable L₂ halo and L_2 NRHO required this velocity discontinuity to converge (or TCM) to achieve



Figure 12. Initial guess for BLT to the 9:2 synodic resonant L_2 NRHO in an ephemeris model, rendered in the (a) Earth-centered inertial frame, (b) Sun- B_1 rotating frame, (c) Earth-Moon rotating frame, and (d) zoomed in view of the DRO in the Earth-Moon rotating frame.

convergence, while the sample transfer to the DRO is achieved without a TCM. A velocity discontinuity representing an insertion maneuver is also included in the DRO and NRHO examples. Table 1 contains a summary of the Δv 's required for the three examples and a comparison between the simplified model from the DSTE and the ephemeris model in FreeFlyer. The transfer to the unstable L_2 halo orbit includes a TCM of 81.9 m/s in the ephemeris model, as compared to the fully ballistic trajectory in the DSTE. The BLT to the DRO includes an insertion maneuver of 140.9 m/s in the ephemeris model, an increase over the 100 m/s computed with the simplified dynamics in the DSTE. Finally, the sample transfer to the L_2 NRHO in the ephemeris model requires a combined dV of 107.7 m/s from a TCM and an insertion maneuver, while the combined total computed in the DSTE is 71 m/s. Overall, the Δv costs of the converged trajectories in the higher-fidelity force model do not diverge significantly from the simplified model used by the DSTE. This investigation presents a method to rapidly construct ballistic lunar transfers with reasonable Δv costs. Further reduction in Δv costs can be found use optimization techniques, as demonstrated by Parrish et al.¹⁸ The solutions constructed in the DSTE can also be used to seed these optimization techniques as an initial guess.

	L_2 Halo		DRO		L_2 NRHO	
Model	BCR4BP	Ephemeris	BCR4BP	Ephemeris	BCR4BP	Ephemeris
TCM Δv [m/s]	_	81.9	_	0.0	_	43.5
Insertion Δv [m/s]	_	_	100.0	140.9	71	64.2
Total [m/s]	0	81.9	100	140.9	71	107.7

Table 1. Summary of Δv 's for BLT examples.

CONCLUDING REMARKS

Trajectory design in multi-body regimes presents unique challenges, particularly during preliminary phases of the design process and generation of an initial guess. To address these challenges, development of software tools to construct initial guesses is critical. This investigation leverages the capabilities of the Deep Space Trajectory Explorer for transfer trajectory design in these sensitive multi-body regimes. Specifically, the DSTE is leveraged to construct ballistic lunar transfers, which employ solar gravity to directly access the lunar vicinity from Earth. The features in DSTE allow users to rapidly create, visualize, and interact with large datasets associated with Poincaré maps. These features are utilized to develop a workflow to generate BLTs to cislunar orbits. This workflow includes leveraging periapsis Poincaré maps and filtering techniques to isolate BLTs of interest and rapidly generate initial guesses to be transitioned to a higher-fidelity model. Three examples are presented that demonstrate the flexibility of the methodology. Ballistic lunar transfers to an unstable halo orbit, a nearly stable NRHO, and a stable DRO are constructed. Then, a process is summarized that exploits the capaibilites of FreeFlyer to transition solutions found in the DSTE to a higher-fidelity model. The process to transition trajectories employs a differential corrections method to ensure that desirable characteristics are maintained when obtaining a trajectory in the Sun-Earth-Moon ephemeris model. This investigation presents an end-to-end methodology using the DSTE to construct ballistic lunar transfers to cislunar orbits and seeks to highlight that effective software tooling streamlines the design process to generate initial guess solutions in this complex dynamical regime.

ACKNOWLEDGEMENTS

The authors would like to thank Stephen Scheuerle for his insight into the ephemeris transition process and to Nate Johnson and Justin Covey for their help with working through the development of new features in the DSTE.

REFERENCES

[1] National Aeronautics and Space Administration, "NASA's Lunar Exploration Program Overview," *NP*-2020-05-2853-HQ, Sept. 2020.



Figure 13. Comparison between the initial guess and converged solution for each of the periodic orbits in the Earth-Moon rotating frame. The initial guesses in the left column are ten stacked periodic orbits. The effects of propagation within the full ephemeris model are apparent due to the distortion of the converged periodic orbits.

- [2] A. Haapala, M. Vaquero, T. Pavlak, K. Howell, and D. Folta, "Trajectory Selection Strategy for Tours in the Earth-Moon System," AAS/AIAA Astrodynamics Specialist Conference, Hilton Head, South Carolina, Aug. 2013.
- [3] D. Guzzetti, N. Bosanac, A. Haapala, K. Howell, and D. Folta, "Rapid Trajectory Design in the Earth-





Figure 14. Converged BLT to an unstable L_2 halo orbit in an ephemeris model, rendered in the (a) Earth-centered inertial frame, (b) Sun- B_1 rotating frame, (c) Earth-Moon rotating frame, and (d) zoomed in view of the periodic orbit in the Earth-Moon rotating frame.

Moon Ephemeris System via an Interactive Catalog of Periodic and Quasi-Periodic Orbits," 66th International Astronautical Congress, Jerusalem, Israel, Oct. 2015.

- [4] M. Vaquero and J. Senent, "Poincare: A Multi-Body, Multi-System Trajectory Design Tool in MONTE," 7th International Conference on Astrodynamics Tools and Techniques (ICATT), Oberpfaffenhofen, Germany, Nov. 2018.
- [5] K. Howell and J. P. Anderson, "Generator User's Guide, Version 3.0.2," techreport IOM AAE-0140-012, July 2001.
- [6] M. Lo, "LTool Version 1.0G Delivery," techreport, Jet Propulsion Laboratory, Sept. 2000.
- [7] D. C. Davis, S. M. Phillips, and B. P. McCarthy, "Periapsis Poincaré Maps for Preliminary Trajectory Design in Planet-Moon Systems," AAS/AIAA Astrodynamics Specialist Conference, Vail, Colorado, Aug. 2015.
- [8] D. C. Davis, S. M. Phillips, and B. P. McCarthy, "Multi-Body Mission Design Using the Deep Space Trajectory Explorer," 26th AAS/AIAA Spaceflight Mechanics Meeting, Napa, California, Feb. 2016.
- [9] D. C. Davis, S. M. Phillips, and B. P. McCarthy, "Trajectory Design for Saturnian Ocean Worlds Orbiters Using Multidimensional Poincaré Maps," *Acta Astronautica*, Vol. 143, Feb. 2018, pp. 16–28.
- [10] S. M. Phillips, "JavaFX 3D: Advanced Application Development," JavaOne Conference, San Francisco, California, Sept. 2014.





Figure 15. Converged BLT to a lunar distant retrograde orbit in an ephemeris model, rendered in the (a) Earth-centered inertial frame, (b) Sun- B_1 rotating frame, (c) Earth-Moon rotating frame, and (d) zoomed in view of the DRO in the Earth-Moon rotating frame.

- [11] E. A. Belbruno and J. K. Miller, "Sun-Perturbed Earth-to-Moon Transfers with Ballistic Capture," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 4, 1993, pp. 770–775.
- [12] R. B. Roncoli and K. K. Fujii, "Mission Design Overview for the Gravity Recovery and Interior Laboraotry (GRAIL) Mission," AAA/AIAA Astrodynamics Specialist Conference, Toronto, Ontario, Aug. 2010.
- [13] T. Gardner, B. Cheetham, A. Forsman, C. Meek, E. Kayser, J. Parker, M. Thompson, T. Latchu, R. Rogers, B. Bryant, and T. Svitek, "CAPSTONE: A CubeSat Pathfinder for the Lunar Gateway Ecosystem," *AIAA/USU Small Satellite Conference*, North Logan, Utah, July 2021.
- [14] Y.-J. Song, Y.-R. Kim, J. Bae, J. i. Park, S. Hong, D. Lee, and D.-K. Kim, "Overview of the Flight Dynamics Subsystem for the Korea Pathfinder Lunar Orbiter Mission," *Aerospace*, Vol. 8, Aug. 2021.
- [15] J. Foust, "Japan's ispace updates design of lunar lander," https://spacenews.com/japans-ispace-updatesdesign-of-lunar-lander/, Aug. 2020.
- [16] J. S. Parker and R. L. Anderson, *Low-Energy Lunar Trajectory Design*. Deep Space Communications and Navigation Series, Pasadena, California: Jet Propulsion Laboratory, July 2013.
- [17] R. J. Whitley, D. C. Davis, L. M. Burke, B. P. McCarthy, R. J. Power, M. J. McGuire, and K. C. Howell, "Earth-Moon Near Rectilinear Halo and Butterfly Orbits for Lunar Surface Exploration," AAS/AIAA Astrodynamics Specialist Conference, Snowbird, Utah, Aug. 2018.



Figure 16. Converged BLT to the 9:2 synodic resonant L_2 NRHO in an ephemeris model, rendered in the (a) Earth-centered inertial frame, (b) Sun- B_1 rotating frame, (c) Earth-Moon rotating frame, and (d) zoomed in view of the DRO in the Earth-Moon rotating frame.

- [18] N. L. Parrish, E. Kayser, S. Udupa, J. S. Parker, B. W. Cheetham, and D. C. Davis, "Survey of Ballistic Lunar Transfers to Near Rectilinear Halo Orbit," AAS/AIAA Astrodynamics Specialist Conference, Portland, Maine, Aug. 2019.
- [19] N. L. Parrish, E. Kayser, S. Udupa, J. S. Parker, B. W. Cheetham, and D. C. Davis, "Ballistic Lunar Transfers to Near Rectilinear Halo Orbit: Operation Considerations," *AIAA SciTech 2020 Forum*, Orlando, Florida, Jan. 2020.
- [20] S. Scheuerle and K. Howell, "Tidal Attributes of Low-energy Transfers in the Earth-Moon-Sun System," AAS/AIAA Astrodynamics Specialist Conference, Charlotte, North Carolina, Aug. 2022.
- [21] B. McCarthy and K. Howell, "Four-body cislunar quasi-periodic orbits and their application to ballistic lunar transfer design," *Advances in Space Research*, Vol. 71, Jan. 2023.
- [22] V. Szebehely, *The Theory of Orbits: The Restricted Problem of Three Bodies*. New York, New York: Academic Press, Inc, 1967.

- [23] S. S. Huang, "Very Restricted Four-Body Problem," techreport NASA TN D-501, NASA Goddard Space Flight Center, Sept. 1960.
- [24] K. K. Boudad, "Disposal Dynamics From the Vicinity of Near Rectilinear Halo Orbits in the Earth-Moon-Sun System," MS Thesis, Purdue University, West Lafayette, Indiana, Dec. 2018.
- [25] S. T. Scheuerle, B. P. McCarthy, and K. C. Howell, "Construction of Ballistic Lunar Transfers Leveraging Dynamical Systems Techniques," AAS/AIAA Astrodynamics Specialist Virtual Conference, Lake Tahoe, California, Aug. 2020.
- [26] A. Batcha, J. Williams, T. F. Dawn, J. P. Gutkowski, M. Widner, S. L. Smallwood, B. J. Killeen, E. C. Williams, and R. E. Harpold, "Artemis 1 Trajectory Design and Optimization," AAS/AIAA Astrodynamics Specialist Conference, Virtual, Aug. 2020.
- [27] C. A. Ocampo, "An Architecture for a Generalized Trajectory Design and Optimization System," International Conference on Libration Points and Missions, Aiguablava, Spain, June 2002.
- [28] S. P. Hughes, R. H. Qureshi, S. D. Cooley, and J. J. Parker, "Verification and Validation of the General Mission Analysis Tool (GMAT)," AIAA SPACE Forum, San Diego, California, Aug. 2014.
- [29] a. solutions, "Freeflyer Version 7.8.0," Nov. 2022.
- [30] C. R. Short, L. Kay-Bunnell, D. Cather, and N. Kinzly, "Revisiting Trajectory Design with STK Astrogator, Part 2," AAS/AIAA Astrodynamics Specialist Conference, Virtual, Aug. 2021.
- [31] J. Englander, "Rapid Preliminary Design of Interplanetary Trajectories Using the Evolutionary Mission Trajectory Generator," *Interational Conference on Astrodynamics Tools and Techniques*, Darmstadt, Germany, Mar. 2016.
- [32] D. C. Folta, C. M. Webster, N. Bosanac, A. D. Cox, D. Guzzetti, and K. C. Howell, "Trajectory Design Tools for Libration and Cislunar Environments," *International Conference on Astrodynamics Tools and Techniques*, Darmstadt, Germany, Mar. 2016.
- [33] B. P. McCarthy, "Transitioning Cislunar Trajectories to a Higher-Fidelity Model Using FreeFlyer," *FreeFlyer Winter Expo*, Virtual, Dec. 2022⁴.
- [34] K. Boudad, Trajectory Design Between Cislunar Space and Sun-Earth Libration Points in a Four-Body Model. Ph.D. Dissertation, Purdue University, West Lafayette, Indiana, May 2022.
- [35] C. H. Acton, Ancillary Data Services of NASA's Navigation and Ancillary Information Facility, Jan. 1996. https://naif.jpl.nasa.gov/naif/.